A $k$-uniform hypergraph $H$ is $\ell$-Hamiltonian saturated, $1 \leq \ell \leq k - 1$, if $H$ does not contain an $\ell$-overlapping Hamiltonian cycle $C^{(k)}_n(\ell)$ but every hypergraph obtained from $H$ by adding one more edge does contain $C^{(k)}_n(\ell)$. Let $\text{sat}(n, k, \ell)$ be the smallest number of edges in an $\ell$-Hamiltonian saturated $k$-uniform hypergraph on $n$ vertices. Clark and Entringer proved in 1983 that $\text{sat}(n, 2, 1) = \lceil \frac{3n}{2} \rceil$ and the second author showed recently that $\text{sat}(n, k, k - 1) = \Theta(n^{k-1})$. In this paper we prove that $\text{sat}(n, k, \ell) = \Theta(n^{\ell})$ for $\ell = 1$ as well as for all $\ell \geq 0.8k$. 